

Scalar Mesons as “Simple” $Q\bar{Q}$ States

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Abstract. The Nijmegen Unitarized Meson Model and its application to scalar mesons is briefly revisited. It is shown that all scalar states up to 1.5 GeV can be described as 3P_0 $q\bar{q}$ states coupled to the OZI-allowed open and closed two-meson channels consisting of pseudoscalar and vector mesons. Crucial are the manifestation of a resonance-doubling phenomenon, typical for strong S -wave decay, and the employment of truly flavor-symmetric coupling constants. Also S -wave meson-meson scattering is thus reasonably well described, without any parameter fit.

INTRODUCTION

The scalar mesons have been posing a serious problem to hadron spectroscopists over the past three decades, since it seems to be impossible to group these particles into standard nonets typical for mesonic $q\bar{q}$ systems. Among the various apparent inconsistencies with standard mesonic states, we should mention the enigmatic light and broad σ meson alias $f_0(400-1200)$, the light and narrow $f_0(980)$ (old S^*) and $a_0(980)$ (old δ), and the excess of experimental candidates to constitute one ground-state scalar nonet. Therefore, a variety of alternative descriptions and mechanisms have been proposed, such as multiquark ($q^2\bar{q}^2$) configurations, glueballs, $K\bar{K}$ molecules, and instanton contributions.

In this talk, we shall demonstrate that no such exotic approaches are needed to obtain a satisfactory description of the scalar-meson sector, provided one works in a unitarized framework such as the Nijmegen unitarized meson model (NUMM) [1]. In particular, the doubling of resonance poles for the ground states as predicted by the NUMM [2,3], which is typical for S -wave scattering channels strongly coupled to confined channels [4], allows for *two complete* scalar nonets, thus accommodating all experimentally observed states up to 1.5 GeV. Such a resonance doubling was recently also observed in Refs. [5,6], employing a revised version of the Helsinki unitarized quark model (HUQM). However, in the latter work this doubling only occurs for some states, which precludes describing, for instance, an as yet to be confirmed light K_0^* (old κ) and the established $f_0(1500)$ resonance within the very

same framework, contrary to the NUMM. We ascribe this failure to the use of coupling constants for the three-meson vertices that are not flavor independent [3,7], thus leading to a breaking of the usual nonet pattern for mesons.

In the following, we shall very briefly review the essence of the NUMM, present the results for the scalar mesons, and make some concluding remarks in a perspective of future work.

NIJMEGEN UNITARIZED MESON MODEL

The basic unitarization philosophy underlying the NUMM stems from the observation that most mesons are resonances, some of which so broad that their very existence seems doubtful, as for example the $f_0(400-1200)$. So it makes no sense to treat such states as stable $q\bar{q}$ systems, even if *a posteriori* hadronic decays are dealt with in perturbation theory. The problem is that such an approach ignores the possible real mass shifts due to strong decay, which, at least in principle, could be of the same order of magnitude as the resonance widths. To make things worse, there is no reason to presume beforehand that the effects of closed thresholds, corresponding to *virtual* two-meson decays or, in diagrammatic language, to *mesonic loops*, are negligible.

In order to meet these objections, in the NUMM the valence $q\bar{q}$ system describing a stable or “bare” meson and the various OZI-allowed two-meson decay channels are treated on an equal footing. To achieve this, use is made of a coupled-channel Schrödinger-type formalism, in which a physical meson is represented by a long state vector, for example in the case of the f_0 meson given by

$$|f_0\rangle = \begin{pmatrix} n\bar{n} & (l=1) \\ s\bar{s} & (l=1) \\ \pi\pi & (l=0) \\ \eta_n\eta_n & (l=0) \\ \eta_s\eta_s & (l=0) \\ KK & (l=0) \\ \rho\rho & (l=0) \\ \rho\rho & (l=2) \\ \omega\omega & (l=0) \\ \omega\omega & (l=2) \\ \phi\phi & (l=0) \\ \phi\phi & (l=2) \\ K^*K^* & (l=0) \\ K^*K^* & (l=2) \end{pmatrix}, \quad \begin{cases} V_{q\bar{q}} = \frac{1}{2}\mu_q\omega^2 r^2 \\ V_{M_1M_2} = 0 \\ V_{qM} = \tilde{g}\frac{r}{r_0}e^{-\frac{1}{2}(\frac{r}{r_0})^2} \end{cases}. \quad (1)$$

Since the f_0 is a scalar isosinglet, we take two P -wave $q\bar{q}$ channels that can mix, coupled to a series of S - and D -wave two-meson channels consisting of pseudoscalar and vector mesons. The mixing takes place via the meson-meson channels to which both $n\bar{n}$ and $s\bar{s}$ couple, i.e., the channels with kaons. Note that n is shorthand for

u or d quark, so η_n and η_s stand for the ideally mixed, non-strange and strange isosinglet pseudoscalar, respectively. In Eq. 1, we have also given the used potentials, which amount to a harmonic oscillator with constant frequency in the $q\bar{q}$ channels, and a peaked function vanishing at the origin for the transitions between $q\bar{q}$ and meson-meson channels, which should mimic the 3P_0 mechanism (see Ref. [1] for reasons and details). No direct interaction between the two $q\bar{q}$ channels is assumed, so the mixing takes place via the mesonic channels that couple to both. Also, no meson-meson or “final-state” interactions are included for simplicity. These assumptions are not strictly necessary, but facilitate the numerical tractability of the equations [1] and, moreover, allow a cleaner view on what are the pure unitarization effects. A possible relaxation of these restrictions will be discussed in the concluding remarks. As to the generic coupling constant \tilde{g} in Eq. (1), it should be noted that it includes phenomenological factors for one-gluon exchange and closed-threshold suppression [1], besides flavor-symmetric coupling constants for the various 3P_0 three-meson vertices [7].

The NUMM has been applied to heavy quarkonia [8], pseudoscalar and vector mesons [1], and scalar mesons [2], with generally good results for the mesonic spectra and meson-meson phase shifts. Especially the predictions in the scalar sector are of a remarkable quality, considering that all model parameters have been previously fixed in a fit to the light and heavy pseudoscalar and vector mesons, without any additional adjustment to the scalars. However, before examining in detail the scalar sector, let us first try to get a feeling for the possible effects of unitarization in a very simple toy model (see also Ref. [9]).

To study qualitatively the influence of mesonic decay on the bare spectrum of a confining $q\bar{q}$ potential, let us consider the two-channel Schrödinger equation

$$\begin{pmatrix} H_{q\bar{q}} & \lambda V_{qM} \\ \lambda V_{qM} & H_{MM} \end{pmatrix} \begin{pmatrix} \Psi_{q\bar{q}} \\ \Psi_{MM} \end{pmatrix} = E \begin{pmatrix} \Psi_{q\bar{q}} \\ \Psi_{MM} \end{pmatrix}. \quad (2)$$

Here, $H_{q\bar{q}}$ contains a confining potential, that is, a harmonic oscillator, H_{MM} is taken to be a free S -wave Hamiltonian, and λV_{qM} simulates the transitions between the $q\bar{q}$ and meson-meson channel through 3P_0 quark-pair creation. In the case that the transition strength $\lambda = 0$, Eq. (2) just yields two disconnected spectra, a discrete one for the bare $q\bar{q}$ state, and a continuous one for the free two-meson system. Once $\lambda \neq 0$, one unique spectrum emanates, which amounts to a number of resonances resulting from the possibility for the confined $q\bar{q}$ system to decay into two mesons. In Fig. 1 [9], we plot the total meson-meson cross section for the case of small λ . We clearly see that there is an evident correspondence between the found peaks and the discrete $q\bar{q}$ energy levels indicated with crosses, the central resonance positions almost exactly coinciding with the bound-state energies. This is a situation one typically would find in atomic physics. However, in hadronic physics the state of affairs is usually very different, where the now large coupling λ reflects the possibility of strong decay. Such a situation is depicted in Fig. 2 [9], where the resonance peaks and bumps are not only at energies quite off the values in the bare

spectrum, but even different in number. It is obvious that, if anything similar were to happen in real hadron spectroscopy, the consequences would be dramatic, since then hardly any inference could be drawn from the physical spectrum concerning the underlying confining $q\bar{q}$ potential. In the following application of the NUMM to scalar mesons, we shall verify that this indeed occurs.

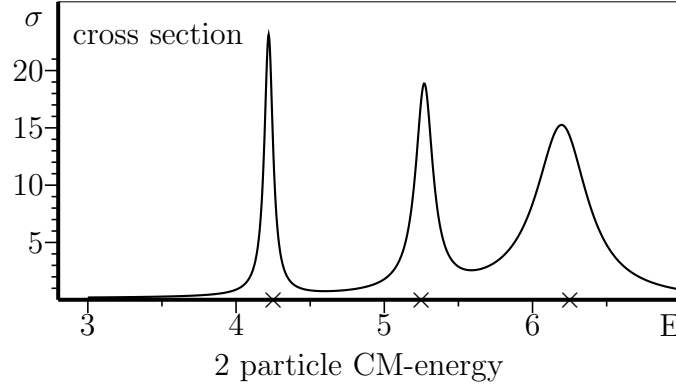


FIGURE 1. Elastic scattering cross section for small transition strength (arbitrary units).
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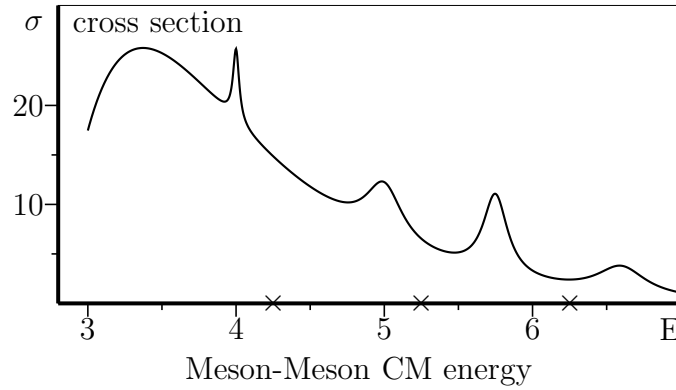


FIGURE 2. Elastic cross section for large transition strength (arbitrary units).
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SCALAR MESONS

As mentioned before, the application of the full NUMM to scalar mesons is straightforward, not involving any additional fit or alteration of the interactions. The only approximation is the omission of color splitting, which we verified to be negligible anyhow, due to the 3P_0 nature of the scalars themselves. The thus obtained S -matrices for all mesonic $J^{PC} = 0^{++}$ states have been searched for poles in the second Riemann sheet. The results for the real parts of the found poles are given in Table 1 [3], together with the predictions of model [5,6] (HUQM), the experimentally confirmed candidates, and the respective interpretations in terms of $q\bar{q}$ states. The most striking feature of the NUMM results is a doubling of states with respect to the bare $q\bar{q}$ spectrum, which was already forboded somehow by the toy model presented above. Here, we restrict ourselves to note that this dynamical phenomenon is typical for S -wave scattering channels strongly coupled to confined channels, and refer to Refs. [3,4] for more details and discussion. The resonance doubling allows for an identification of all observed scalar states up to 1.5 GeV, even obtaining one extra resonance not yet confirmed by experiment, namely a light K_0^* (old κ). But also this state has recently received renewed phenomenological and theoretical support [11–15]. As to the model of Ref. [5,6], we observe from Table 1 that such a resonance is not predicted, nor the established $f_0(1500)$. Moreover, the $f_0(1300)$ is interpreted as mainly $s\bar{s}$, while we claim it is predominantly $n\bar{n}$ and the $f_0(1500)$ mainly $s\bar{s}$. Apart from considering our interpretation more natural and favored by the known decay rates [3], we should mention a very recent lattice calculation largely supporting our model prediction [16]. The fact that model [5,6] fails to find two complete scalar nonets we ascribe to the use of coupling constants for the three-meson vertices that are not flavor independent [3]. Here, the crucial point is a point-particle approach in the derivation of the couplings, which leads to a wrong normalization in the case of the scalar mesons, being 3P_0 states themselves just as the created $q\bar{q}$ pairs [7,17].

Another feature of the NUMM, and also of the HUQM for that matter, is the automatic obtainment of a unitary, analytic S -matrix, which allows for a straightforward calculation of partial cross sections and phase shifts. Thus, we present our results for the elastic S -wave $\pi\pi$ and $K\pi$ phase shifts, in Figs. 3 and 4, respectively. We must reemphasize that these are model *predictions* and not the result of a fit. In that perspective, the results are surprisingly good, reproducing the bulk features of the experimental phases, including the resonant structures. In the $\pi\pi$ case, both the broad structure from roughly 400 to 950 MeV, owing to a $f_0(400-1200)$ (or σ) pole at $470 - 208i$ MeV, and the sharp resonance close to 1 GeV, due to a $f_0(980)$ pole at $994 - 17i$ MeV, are reasonably well described. Of course, some background structure is clearly lacking, which is no surprise in view of the neglect of final-state interactions in the meson-meson channels. Also notice that in the $K\pi$ case, where final-state interactions from t -channel meson exchanges are expected to be less important, the phase shifts are extremely well reproduced in the energy region 0.7–1.2 GeV, exactly where we find the lowest K_0^* pole, i.e., at $727 - 263i$ MeV. So

TABLE 1. Scalar-meson predictions and $q\bar{q}$ interpretations for the HUQM and NUMM, together with experimentally established states.

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Resonance	HUQM [5,6]		NUMM [2,3]		Exp. [10]
	$\text{Re}E_{\text{pole}}$	$q\bar{q}$ configuration	$\text{Re}E_{\text{pole}}$	$q\bar{q}$ configuration	Mass
$\sigma/f_0(400-1200)$	470	$1^{st} \approx \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$	470	$1^{st} \approx \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$	400-1200
$S^*/f_0(980)$	1006	$1^{st} \approx s\bar{s}$	994	$1^{st} \approx s\bar{s}$	980 ± 10
$\delta/a_0(980)$	1094	$1^{st} \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$	968	$1^{st} \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$	983 ± 1
κ/K_0^*	-	-	727	$1^{st} s\bar{d}$?
$f_0(1370)$	1214	$2^{nd} \approx s\bar{s}$	1300	$2^{nd} \approx \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$	1200–1500
$f_0(1500)$	-	-	1500	$2^{nd} \approx s\bar{s}$	1500 ± 10
$a_0(1450)$	1592	$2^{nd} \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$	1300	$2^{nd} \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$	1474 ± 19
$K_0^*(1430)$	1450	$1^{st} s\bar{d}$	1400	$2^{nd} s\bar{d}$	1429 ± 6

it is unmistakably demonstrated that this very controversial resonance is perfectly compatible with a non-resonant behavior of the phases in the same region.

CONCLUDING REMARKS

In the foregoing, we hope to have made it clear that, for a reliable and detailed description of meson spectra in general, the effects of mesonic loops and decay should be taken into account. In particular, this is *a fortiori* true in the case of the scalar mesons, where, from a theoretical point of view, one is faced with very large couplings to S -wave two-meson channels, and, on the experimental side, a confusing picture of many broad as well as narrow resonances of very disparate masses emerges. Whatever the used approach, however, extreme care is required in the choice of the classes of the to be included decay channels, and in the computation of the respective coupling constants, lest one introduce explicitly flavor-breaking mechanisms that may distort the spectra in an unrealistic fashion.

The NUMM employed here may, of course, be subject to improvements. As indicated before, final-state interactions from t -channel meson exchanges should be included, aiming at restoring crossing symmetry to some degree, and in order to allow for a more accurate reproduction of the experimental meson-meson phase shifts. Furthermore, relativity should be addressed in a thorough way than just by some relativistic kinematics, preferably in a covariant quasipotential framework. This would require a profound overhaul of the mathematical formulation of the model, but could then make further refinements of the used interactions feasible.

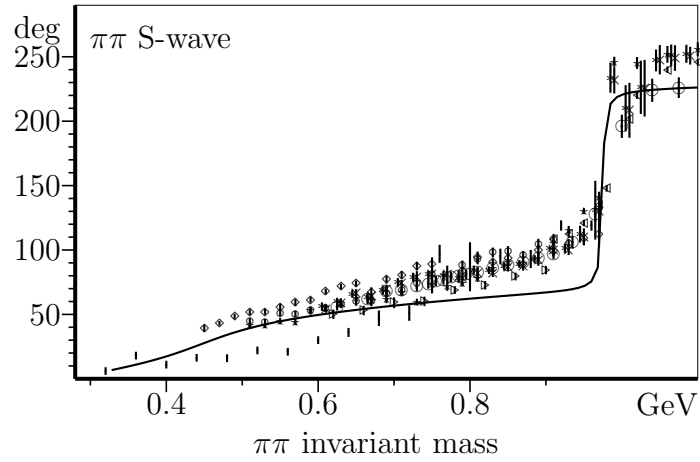


FIGURE 3. Model [2] results for $\pi\pi$ elastic S -wave phase shifts. The various sets of data are taken from (\odot , [18]), ($*$, [19]), (\star , \times , \diamond , \triangleleft , \triangleright respectively for analyses A, B, C, D, and E of [20]), (\circ , [21]), and (\cdot , [22]).

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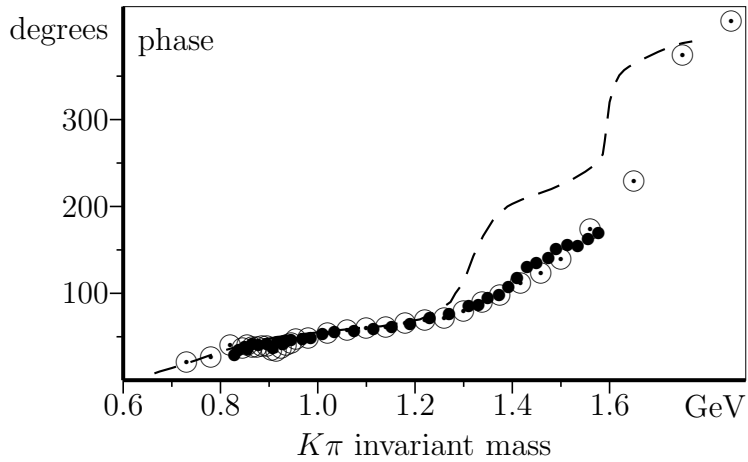


FIGURE 4. Kaon-pion $I = \frac{1}{2}$ S -wave phase shifts. The data indicated by \odot are taken from Ref. [23] and by \bullet from Ref. [24]. The model results (dashed line) are taken from Ref. [2].

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